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Charge neutrality effects on two-flavor color superconductivity

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Four different phases of the two-flavor quark system, i.e., the normal quark matter and color superconducting phases with and without the charge neutrality condition, are investigated in the framework of the SU(2) Nambu–Jona-Lasinio model. It is found that the color superconducting phase without charge neutrality has the lowest thermodynamic potential, and the charge neutral systems have higher thermodynamic potentials. However, the BCS pairing always lowers the system's thermodynamic potential, i.e., for both the charged and charge neutral systems, the superconducting phases always have lower thermodynamic potentials. Compared with the BCS gap Δ_0 for the color superconducting phase without charge neutrality, the BCS gap Δ in the charge neutral color superconducting phase has been largely reduced. When the thermodynamic potential for the charge neutral color superconducting phase equals that in the neutral normal quark matter, the diquark condensate disappears.

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I. INTRODUCTION

Because of the existence of an attractive interaction in the antitriplet quark-quark channel in QCD, the cold and dense quark matter has been believed to favor the formation of the diquark condensate and being in the superconducting phase [1–4].

The QCD phase structure at high baryon density is determined by how many kinds of quarks can participate in the pairing. If we have an ideal system with two massless quarks *u* and *d*, and the three colored *u* quarks and three colored *d* quarks have the same Fermi surface, the system will be in the two-flavor color superconducting phase (2SC); and if we have an ideal system with three massless quarks *u*, *d*, and *s*, and all the nine quarks have the same Fermi surface, then all the quarks will participate in pairing and form a color-flavor-locking (CFL) phase [5].

In the real world, the strange quark mass m_s should be considered, which will reduce the strange quark's Fermi surface [6]. If m_s is large enough, there will be no pairing between us and ds, and a $2\,\mathrm{SC}+s$ phase will be favored.

Also, in reality, the system should be electric and color neutral, the existence of electrons in the system will shift the Fermi surface of the two pairing quarks, and the system can be in the BCS phase or the crystalline phase [7] or normal quark matter, depending on how large the difference in the chemical potentials of the two pairing quarks is.

Color neutrality was first investigated in [8]. In a recent paper [9] it was argued that the $2\,\mathrm{SC}+s$ phase would not appear if the electric and color charge neutrality condition was added, which also agreed with the results in [10] by using the SU(3) Nambu–Jona-Lasinio (NJL) model and taking into account the self-consistent effective quark mass $m_f(\mu)$, where f refers to u,d,s. It is found in [10] that the charge neutrality has a large effect on the s quark mass, i.e., $m_s(\mu)$ begins to decrease at a smaller $\mu \simeq 400$ MeV than the case if no charge neutrality is considered, such as in [11,12], where $m_s(\mu)$ begins to decrease at about $\mu \simeq 550$ MeV.

In this paper, complementary to [10], we investigate the effect of charge neutrality on two-flavor color superconductivity assuming that there is no strange quark involved in the chemical potential regime $\mu < 550$ MeV.

This paper is organized as follows: in Sec. II we extend our method in [13] to derive the thermodynamic potential when charge neutrality is considered; the gap equations and charge neutrality conditions will be derived in Sec. III, and in Sec. IV we will give our numerical results; the conclusions and discussions will be given in Sec. V.

II. THERMODYNAMIC POTENTIAL

A. The Lagrangian

Assuming that the strange quark does not appear in the system, we use the SU(2) Nambu–Jona-Lasinio model, and only consider scalar and pseudoscalar mesons and scalar diquarks. The Lagrangian density has the form as

$$\mathcal{L}=\bar{q}(i\gamma^{\mu}\partial_{\mu}-m_0)q+G_S[(\bar{q}q)^2+(\bar{q}i\gamma_5\vec{\tau}q)^2] + G_D[(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)], \tag{1}$$

where $q^C=C\bar{q}^T$, $\bar{q}^C=q^TC$ are charge-conjugate spinors, $C=i\gamma^2\gamma^0$ is the charge conjugation matrix (the superscript T denotes the transposition operation), the quark field $q\equiv q_{i\alpha}$ with $i=1,2$ and $\alpha=1,2,3$ is a flavor doublet and color triplet, as well as a four-component Dirac spinor, $\vec{\tau}=(\tau^1,\tau^2,\tau^3)$ are Pauli matrices in the flavor space, and $(\varepsilon)^{ik}\equiv\varepsilon^{ik}$, $(\epsilon^b)^{\alpha\beta}$

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$\equiv\epsilon^{\alpha\beta b}$ are antisymmetric tensors in the flavor and color spaces, respectively. G_S and G_D are independent effective coupling constants in the scalar quark-antiquark and scalar diquark channels.

After bosonization, one can obtain the linearized version of the model (1)

$$\tilde{\mathcal{L}}=\bar{q}(i\gamma^\mu\partial_\mu-m)q-\frac{1}{2}\Delta^{*b}(i\bar{q}^C\varepsilon\epsilon^b\gamma_5 q)-\frac{1}{2}\Delta^b(i\bar{q}\varepsilon\epsilon^b\gamma_5 q^C)-\frac{\sigma^2}{4G_S}-\frac{\Delta^{*b}\Delta^b}{4G_D}, \tag{2}$$

where we have assumed that there will be no pion condensate and introduced the constituent quark mass

$$m=m_0+\sigma. \tag{3}$$

From general considerations, there should be eight scalar diquark condensates [14,15]. In the case of the NJL type model, the diquark condensates related to the momentum vanish, and there is only one 0^+ diquark gap with the Dirac structure $\Gamma=\gamma_5$ for the massless quark, and another 0^+ diquark condensate with Dirac structure $\Gamma=\gamma_0\gamma_5$ at nonzero quark mass. In this paper we assume that the contribution of the diquark condensate with $\Gamma=\gamma_0\gamma_5$ is small, and we only consider the diquark condensate with $\Gamma=\gamma_5$. The diquark condensate with Dirac structure $\gamma_5\gamma_0$ has been recently discussed in [16].

We can choose that the diquark condenses in the third color direction, i.e., only the first two colors participate in the condensate, while the third one does not.

The model is nonrenormalizable, and a momentum cutoff Λ should be introduced. The parameters G_S and Λ in the chiral limit $m_0=0$ are fixed as

$$G_S=5.0163\ \ \mathrm{GeV}^{-2},\Lambda=0.6533\ \ \mathrm{GeV}. \tag{4}$$

The corresponding effective mass $m=0.314$ GeV, and we will choose $G_D=3/4G_S$ in our numerical calculations.

B. Partition function and thermodynamic potential

The partition function of the grand canonical ensemble can be evaluated by using the standard method [17,18],

$$\mathcal{Z}=N'\int[d\bar{q}][dq]\exp\left\{\int_0^\beta d\tau\int d^3\vec{x}(\tilde{\mathcal{L}}+\mu\bar{q}\gamma_0 q)\right\}, \tag{5}$$

where $\beta=1/T$ is the inverse of temperature T, and μ is the chemical potential. When electric and color charge neutrality is considered, the chemical potential μ is a diagonal 6×6 matrix in flavor and color space, and can be expressed as

$$\mu=\mathrm{diag}(\mu_1,\mu_2,\mu_3,\mu_4,\mu_5,\mu_6), \tag{6}$$

where μ_1,μ_2,μ_3 are for the three colored u quarks, and μ_4,μ_5,μ_6 are for the three colored d quarks.

Like in [9,10], the chemical potential for each color and flavor quark is specified by its electric and color charges $\mu_i=\mu-Q_e\mu_e+T_3\mu_{3c}+T_8\mu_{8c}$, where Q_e, T_3, and T_8 are generators of $U(1)_Q$, $U(1)_3$, and $U(1)_8$. Because the diquark condenses in the third color direction, and the first two colored quarks are degenerate, we can assume $\mu_{3c}=0$. For the same flavor, the difference of chemical potentials between the first two colored quarks and the third colored quark is induced by μ_{8c}, and for the same color, the difference of chemical potentials between u and d is induced by μ_e.

The explicit expressions for each color and flavor quark's chemical potential are

$$\mu_1=\mu_2=\mu-\frac{2}{3}\mu_e+\frac{1}{3}\mu_{8c},$$

$$\mu_4=\mu_5=\mu+\frac{1}{3}\mu_e+\frac{1}{3}\mu_{8c},$$

$$\mu_3=\mu-\frac{2}{3}\mu_e-\frac{2}{3}\mu_{8c},$$

$$\mu_6=\mu+\frac{1}{3}\mu_e-\frac{2}{3}\mu_{8c}. \tag{7}$$

For the convenience of calculations, we define the mean chemical potential $\bar{\mu}$ for the pairing quarks q_1,q_5, and q_2,q_4

$$\bar{\mu}=\frac{\mu_1+\mu_5}{2}=\frac{\mu_2+\mu_4}{2}=\mu-\frac{1}{6}\mu_e+\frac{1}{3}\mu_{8c}, \tag{8}$$

and the difference of the chemical potential $\delta\mu$

$$\delta\mu=\frac{\mu_5-\mu_1}{2}=\frac{\mu_4-\mu_2}{2}=\mu_e/2. \tag{9}$$

Because the third colored u and d, i.e., the third and sixth quarks do not participate in the diquark condensate, the partition function can be written as a product of three parts,

$$\mathcal{Z}=\mathcal{Z}_{const}\mathcal{Z}_{36}\mathcal{Z}_{15,24}. \tag{10}$$

The constant part is

$$\mathcal{Z}_{const}=N'\exp\left\{-\int_0^\beta d\tau\int d^3\vec{x}\left[\frac{\sigma^2}{4G_S}+\frac{\Delta^*\Delta}{4G_D}\right]\right\}. \tag{11}$$

The $\mathcal{Z}_{36}$ part is for the unpairing quarks q_3 (u_3) and the q_6 (d_3), and $\mathcal{Z}_{15,24}$ part is for the quarks participating in pairing, q_1 (u_1) paired with q_5 (d_2), and q_2 (u_2) paired with q_4 (d_1). In the following two sections, we will derive the contributions of $\mathcal{Z}_{36}$ and $\mathcal{Z}_{15,24}$.

1. Calculation of $\mathcal{Z}_{36}$

Introducing the 8-spinors for q_3 and q_6,

$$\bar{\Psi}_{36}=(\bar{q}_3,\bar{q}_6;\bar{q}_3^C,\bar{q}_6^C), \tag{12}$$

we can express $\mathcal{Z}_{36}$ as

$$\mathcal{Z}_{36}=\int[d\Psi_{36}]\exp\left\{\frac{1}{2}\sum_{n,\vec{p}}\bar{\Psi}_{36}\frac{[G_0^{-1}]_{36}}{T}\Psi_{36}\right\}$$
$$=\mathrm{Det}^{1/2}(\beta[G_0^{-1}]_{36}), \tag{13}$$

where the determinantal operation Det is to be carried out over the Dirac, color, flavor, and the momentum-frequency space, and $[G_0^{-1}]_{36}$ has the form of

$$[G_0^{-1}]_{36}=\begin{pmatrix}[G_0^+]_3^{-1} & 0 & 0 & 0\\ 0 & [G_0^+]_6^{-1} & 0 & 0\\ 0 & 0 & [G_0^-]_3^{-1} & 0\\ 0 & 0 & 0 & [G_0^-]_6^{-1}\end{pmatrix}, \tag{14}$$

with

$$[G_0^{\pm}]_i^{-1}=\not{p}\pm\not{\mu}_i-m. \tag{15}$$

Here we have used $\not{p}=p_\mu\gamma^\mu$ and $\not{\mu}_i=\mu_i\gamma_0$.

For the two quarks not participating in the diquark condensate, from Eq. (13), we can have

$$\ln\mathcal{Z}_{36}=\frac{1}{2}\ln\mathrm{Det}(\beta[G_0^{-1}]_{36})$$
$$=\frac{1}{2}\ln\{\mathrm{Det}(\beta[G_0^+]_3^{-1}\}\{\mathrm{Det}(\beta[G_0^-]_3^{-1})\}$$
$$\times\{\mathrm{Det}(\beta[G_0^+]_6^{-1})\mathrm{Det}(\beta[G_0^-]_6^{-1})\}. \tag{16}$$

We first work out

$$\{\mathrm{Det}(\beta[G_0^+]_3^{-1})\mathrm{Det}(\beta[G_0^-]_3^{-1})\}=\beta^4[p_0^2-E_3^{+\,2}][p_0^2-E_3^{-\,2}],$$
$$\{\mathrm{Det}(\beta[G_0^+]_6^{-1})\mathrm{Det}(\beta[G_0^-]_6^{-1})\}=\beta^4[p_0^2-E_6^{+\,2}][p_0^2-E_6^{-\,2}], \tag{17}$$

with $E_3^{\pm}=E\pm\mu_3$ and $E_6^{\pm}=E\pm\mu_6$ where $E=\sqrt{\mathbf{p}^2+m^2}$. Considering the determinant in the flavor, color, spin spaces, and momentum-frequency space, we get the expression

$$\ln\mathcal{Z}_{36}=\sum_n\sum_{\vec{p}}\{\ln(\beta^2[p_0^2-E_3^{+\,2}])+\ln(\beta^2[p_0^2-E_3^{-\,2}])$$
$$+\ln(\beta^2[p_0^2-E_6^{+\,2}])+\ln(\beta^2[p_0^2-E_6^{-\,2}])\}. \tag{18}$$

2. Calculation of $\mathcal{Z}_{15,24}$

The calculation of $\mathcal{Z}_{15,24}$ here is much more complicated than that when the two pairing quarks have the same Fermi surface [13]. Also, we introduce the Nambu-Gokov formalism for q_1,q_2,q_4 and q_5, i.e.,

$$\bar{\Psi}=(\bar{q}_1,\bar{q}_2,\bar{q}_4,\bar{q}_5;\bar{q}_1^C,\bar{q}_2^C,\bar{q}_4^C,\bar{q}_5^C). \tag{19}$$

The $\mathcal{Z}_{15,24}$ can have the simple form as

$$\mathcal{Z}_{15,24}=\int[d\Psi]\exp\left\{\frac{1}{2}\sum_{n,\vec{p}}\bar{\Psi}\frac{G_{15,24}^{-1}}{T}\Psi\right\}$$
$$=\mathrm{Det}^{1/2}(\beta[G^{-1}]_{15,24}), \tag{20}$$

where

$$G_{15,24}^{-1}=\begin{pmatrix}[G_0^+]_{15,24}^{-1} & \Delta^-\\ \Delta^+ & [G_0^-]_{15,24}^{-1}\end{pmatrix}, \tag{21}$$

with

$$[G_0^{\pm}]_{15,24}^{-1}=\begin{pmatrix}[G_0^{\pm}]_1^{-1} & 0 & 0 & 0\\ 0 & [G_0^{\pm}]_2^{-1} & 0 & 0\\ 0 & 0 & [G_0^{\pm}]_4^{-1} & 0\\ 0 & 0 & 0 & [G_0^{\pm}]_5^{-1}\end{pmatrix}, \tag{22}$$

and the matrix form for $\Delta^{\pm}$ is

$$\Delta^-=-i\Delta\gamma_5\begin{pmatrix}0 & 0 & 0 & 1\\ 0 & 0 & -1 & 0\\ 0 & -1 & 0 & 0\\ 1 & 0 & 0 & 0\end{pmatrix},\quad \Delta^+=\gamma^0(\Delta^-)^\dagger\gamma^0. \tag{23}$$

From Eq. (20), we have

$$\ln Z_{15,24}=\frac{1}{2}\ln\mathrm{Det}(\beta G_{15,24}^{-1}). \tag{24}$$

For a 2×2 matrix with elements A, B, C, and D, we have the identity

$$\mathrm{Det}\begin{pmatrix}A & B\\ C & D\end{pmatrix}=\mathrm{Det}(-CB+CAC^{-1}D)$$
$$=\mathrm{Det}(-BC+BDB^{-1}A). \tag{25}$$

Replacing A, B, C, and D with the corresponding elements of $G_{15,24}^{-1}$, we have

$$\mathrm{Det}(G_{15,24}^{-1})=\mathrm{Det}\,D_1=\mathrm{Det}(-\Delta^+\Delta^-$$
$$+\Delta^+[G_0^+]_{15,24}^{-1}[\Delta^-]^{-1}[G_0^-]_{15,24}^{-1})$$
$$=\mathrm{Det}\,D_2=\mathrm{Det}(-\Delta^-\Delta^++\Delta^-[G_0^-]_{15,24}^{-1}$$
$$\times[\Delta^+]^{-1}[G_0^+]_{15,24}^{-1}). \tag{26}$$

Using the massive energy projectors $\Lambda_{\pm}$ in [13] for each flavor and color quark, we can work out the diagonal matrix D_1 and D_2 as

$$(D_1)_{11}=(D_1)_{22}=[(p_0+\delta\mu)^2-[\overline{E_\Delta^-}]^2]\Lambda_-+[(p_0+\delta\mu)^2$$
$$-[\overline{E_\Delta^+}]^2]\Lambda_+,$$

$$(D_1)_{33}=(D_1)_{44}=[(p_0-\delta\mu)^2-[\overline{E_\Delta^-}]^2]\Lambda_-+[(p_0-\delta\mu)^2-[\overline{E_\Delta^+}]^2]\Lambda_+,$$

$$(D_2)_{11}=(D_2)_{22}=[(p_0-\delta\mu)^2-[\overline{E_\Delta^+}]^2]\Lambda_-+[(p_0-\delta\mu)^2-[\overline{E_\Delta^-}]^2]\Lambda_+,$$

$$(D_2)_{33}=(D_2)_{44}=[(p_0+\delta\mu)^2-[\overline{E_\Delta^+}]^2]\Lambda_-+[(p_0+\delta\mu)^2-[\overline{E_\Delta^-}]^2]\Lambda_+, \tag{27}$$

where $\overline{E_\Delta^\pm}=\sqrt{(E\pm\bar{\mu})^2+\Delta^2}$.

With the above equations, Eq. (24) can be expressed as

$$\ln \mathcal{Z}_{15,24}=2\sum_n\sum_p (\ln[\beta^2[p_0^2-(\overline{E_\Delta^-}+\delta\mu)]^2\}+\ln\{\beta^2[p_0^2-(\overline{E_\Delta^-}-\delta\mu)]^2\}+\ln\{\beta^2[p_0^2-(\overline{E_\Delta^+}+\delta\mu)]^2\}+\ln\{\beta^2[p_0^2-(\overline{E_\Delta^+}-\delta\mu)]^2\}). \tag{28}$$

C. The thermodynamic potential

Using the helpful relation

$$\ln \mathcal{Z}_f=\sum_n \ln[\beta^2(p_0^2-E_p^2)]=\beta[E_p+2T\ln(1+e^{-\beta E_p})], \tag{29}$$

we can evaluate the thermodynamic potential of the quark system

$$\Omega_q=\frac{m^2}{4G_S}+\frac{\Delta^2}{4G_D}-2\int\frac{d^3p}{(2\pi)^3}\{2E+T\times\ln[1+\exp(-\beta E_3^+)]+T\ln[1+\exp(-\beta E_3^-)]+T\ln[1+\exp(-\beta E_6^+)]+T\ln[1+\exp(-\beta E_6^-)]+2\overline{E_\Delta^+}+2\overline{E_\Delta^-}+2T\ln[1+\exp(-\beta\overline{E_{\Delta^+}^+})]+2T\times\ln[1+\exp(-\beta\overline{E_{\Delta^-}^+})]+2T\ln[1+\exp(-\beta\overline{E_{\Delta^+}^-})]+2T\ln[1+\exp(-\beta\overline{E_{\Delta^-}^-})]\} \tag{30}$$

with $\overline{E_{\Delta^\pm}^\pm}=\overline{E_\Delta^\pm}\pm\delta\mu$.

For the total thermodynamic potential, we should include the contribution from the electron gas, Ω_e. Assuming the electron's mass is zero, we have

$$\Omega_e=-\frac{\mu_e^4}{12\pi^2}. \tag{31}$$

The total thermodynamic potential of the system is

$$\Omega=\Omega_q+\Omega_e. \tag{32}$$

III. GAP EQUATIONS AND CHARGE NEUTRALITY CONDITION

From the thermodynamic potential Eq. (32), we can derive the gap equations of the order parameters m and Δ for the chiral and color superconducting phase transitions.

A. Gap equation for the quark mass

The gap equation for the quark mass can be derived by using

$$\frac{\partial\Omega}{\partial m}=0. \tag{33}$$

The explicit expression for the above equation is

$$m\left[1-4G_S\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{E}\left[2\frac{\overline{E^-}}{\overline{E_\Delta^-}}[1-\tilde{f}(\overline{E_{\Delta^+}^-})-\tilde{f}(\overline{E_{\Delta^-}^-})]+2\frac{\overline{E^+}}{\overline{E_\Delta^+}}[1-\tilde{f}(\overline{E_{\Delta^+}^+})-\tilde{f}(\overline{E_{\Delta^-}^+})]+[2-\tilde{f}(E_3^+)-\tilde{f}(E_3^-)-\tilde{f}(E_6^+)-\tilde{f}(E_6^-)]\right]\right]=0, \tag{34}$$

with the Fermi distribution function $\tilde{f}(x)=1/[\exp\{\beta x\}+1]$. $m=0$ corresponds to the chiral symmetric phase; $m\neq 0$ corresponds to the chiral symmetry breaking phase.

B. Gap equation for the diquark condensate

$$\frac{\partial\Omega}{\partial\Delta}=0, \tag{35}$$

we can derive the gap equation for diquark condensate

$$\Delta\left[1-4G_D\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left[2\frac{1}{\overline{E_\Delta^-}}[1-\tilde{f}(\overline{E_{\Delta^+}^-})-\tilde{f}(\overline{E_{\Delta^-}^-})]+2\frac{1}{\overline{E_\Delta^+}}[1-\tilde{f}(\overline{E_{\Delta^+}^+})-\tilde{f}(\overline{E_{\Delta^-}^+})]\right]\right]=0. \tag{36}$$

C. Number density for each color and flavor

The number densities for each color and flavor can be derived using the thermodynamic potential in terms of μ_i. Because there still exists a $SU(2)_c$ symmetry, the first two colored up quarks are degenerate, and the first two colored down quarks are also degenerate, i.e., the number densities for the first two colored up quarks are the same,

$$n_1=n_2=\int\frac{d^3\mathbf{p}}{(2\pi)^3}\Bigg[\frac{\overline{E}^+}{\overline{E}_\Delta^+}[1-\tilde{f}(\overline{E}_{\Delta+}^+)-\tilde{f}(\overline{E}_{\Delta-}^+)] -\frac{\overline{E}^-}{\overline{E}_\Delta^-}[1-\tilde{f}(\overline{E}_{\Delta+}^-)-\tilde{f}(\overline{E}_{\Delta-}^-)]+\tilde{f}(\overline{E}_{\Delta+}^-)+\tilde{f}(\overline{E}_{\Delta+}^+) -\tilde{f}(\overline{E}_{\Delta-}^+)-\tilde{f}(\overline{E}_{\Delta-}^-)\Bigg], \tag{37}$$

and the first two colored down quarks also have the same number densities,

$$n_4=n_5=\int\frac{d^3\mathbf{p}}{(2\pi)^3}\Bigg[\frac{\overline{E}^+}{\overline{E}_\Delta^+}[1-\tilde{f}(\overline{E}_{\Delta+}^+)-\tilde{f}(\overline{E}_{\Delta-}^+)] -\frac{\overline{E}^-}{\overline{E}_\Delta^-}[1-\tilde{f}(\overline{E}_{\Delta+}^-)-\tilde{f}(\overline{E}_{\Delta-}^-)]-\tilde{f}(\overline{E}_{\Delta+}^-)-\tilde{f}(\overline{E}_{\Delta+}^+) +\tilde{f}(\overline{E}_{\Delta-}^+)+\tilde{f}(\overline{E}_{\Delta-}^-)\Bigg]. \tag{38}$$

However, in principle, it is not necessary for the paired two quarks such as u_1 and d_2 or u_2 and d_1 to have same number densities,

$$n_1-n_5=n_2-n_4=2\int\frac{d^3\mathbf{p}}{(2\pi)^3}[\tilde{f}(\overline{E}_{\Delta+}^-)+\tilde{f}(\overline{E}_{\Delta+}^+)-\tilde{f}(\overline{E}_{\Delta-}^+) -\tilde{f}(\overline{E}_{\Delta-}^-)]. \tag{39}$$

Only in the case of $T=0$ and $\delta\mu<\Delta$ can we have equal number densities for the two paired quarks, and thus $n_1=n_5=n_2=n_4$.

But for the unpaired u_3 and d_3, because of the mismatch of the Fermi momentum, they have different number densities. For u_3, the number density is

$$n_3=2\int\frac{d^3\mathbf{p}}{(2\pi)^3}[-\tilde{f}(E_3^+)+\tilde{f}(E_3^-)], \tag{40}$$

and d_3 has the number density

$$n_6=2\int\frac{d^3\mathbf{p}}{(2\pi)^3}[-\tilde{f}(E_6^+)+\tilde{f}(E_6^-)]. \tag{41}$$

D. Color charge neutrality

The color charge neutrality condition is to choose μ_{8c} such that the system has zero net charge T_8, i.e.,

$$n_8=-\frac{\partial\Omega}{\partial\mu_{8c}}=0. \tag{42}$$

Evaluating the above equation, we have the color charge neutrality condition as

$$\int\frac{d^3\mathbf{p}}{(2\pi)^3}\Bigg[-\frac{\overline{E}^-}{\overline{E}_\Delta^-}[1-\tilde{f}(\overline{E}_{\Delta+}^-)-\tilde{f}(\overline{E}_{\Delta-}^-)]+\frac{\overline{E}^+}{\overline{E}_\Delta^+}[1-\tilde{f}(\overline{E}_{\Delta+}^+) -\tilde{f}(\overline{E}_{\Delta-}^+)]+[\tilde{f}(E_3^+)-\tilde{f}(E_3^-)]+[\tilde{f}(E_6^+)-\tilde{f}(E_6^-)]\Bigg]=0. \tag{43}$$

E. Electric charge neutrality

Similarly, the electric charge neutrality condition is to choose μ_e such that the system has zero net electric charge Q_e, i.e.,

$$n_Q=-\frac{\partial\Omega}{\partial\mu_e}=0. \tag{44}$$

From the above equation, we obtain

$$\int\frac{d^3\mathbf{p}}{(2\pi)^3}\Bigg[2\frac{\overline{E}^-}{\overline{E}_\Delta^-}[1-\tilde{f}(\overline{E}_{\Delta+}^-)-\tilde{f}(\overline{E}_{\Delta-}^-)]-2\frac{\overline{E}^+}{\overline{E}_\Delta^+}[1-\tilde{f}(\overline{E}_{\Delta+}^+) -\tilde{f}(\overline{E}_{\Delta-}^+)]-6[\tilde{f}(\overline{E}_{\Delta+}^-)+\tilde{f}(\overline{E}_{\Delta+}^+)-\tilde{f}(\overline{E}_{\Delta-}^+)-\tilde{f}(\overline{E}_{\Delta-}^-)] +4[\tilde{f}(E_3^+)-\tilde{f}(E_3^-)]-2[\tilde{f}(E_6^+)-\tilde{f}(E_6^-)]\Bigg]+\frac{\mu_e^3}{\pi^2}=0. \tag{45}$$

IV. NUMERICAL RESULTS

In the numerical results we investigate the following four different cases: (1) no color superconducting phase, no charge neutrality condition; (2) no color superconducting phase, electric charge neutrality condition added; the electron's chemical potential in this phase will be characterized by μ_e^0; (3) the color superconducting phase exists, no charge neutrality condition; the diquark condensate in this phase will be characterized by Δ_0; (4) the color superconducting phase exists, electric and color charge neutrality condition added; the electron's chemical potential and diquark condensate in this phase will be characterized by μ_e and Δ, respectively.

Figure 1 is for the chiral phase transition in the case of no electric neutrality considered, which is familiar to all of us; the quark possesses a large constituent mass in the low baryon density regime due to chiral symmetry breaking, and restores chiral symmetry at high baryon density.

Figure 2 is the chiral phase diagram when the electric charge neutrality is considered; the solid squares and the solid circles are for quark mass m and the electron's chemical potential μ_e^0, respectively. It is found that in the chiral symmetric phase, there is a large Fermi surface for electrons, and the electron's chemical potential μ_e^0 increases linearly with the increase of the quark's chemical potential μ.

Figure 3 is the phase diagram for color superconductivity without charge neutrality constraints; the quark mass m (squares) and diquark condensate Δ_0 (circles) are plotted as

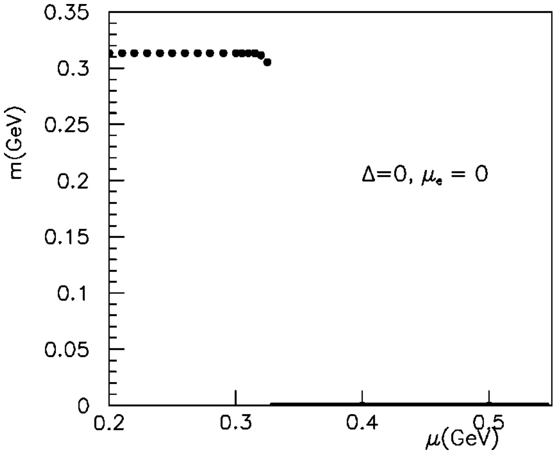

FIG. 1. The quark mass m as a function of chemical potential μ in the case of the no color superconducting phase and the no charge neutrality condition.

functions of the quark's chemical potential μ. It is found that in the color superconducting phase, the magnitude of the diquark condensate Δ_0 is about 100 MeV by using the parameter $G_D = 3/4 G_S$. Δ_0 first increases with increasing μ, and this tendency stops at about $\mu = 530$ MeV, which is related to the momentum cutoff Λ.

Figure 4 is the phase diagram for color superconductivity when electric and color charge neutrality condition is considered; the quark mass m (squares), the diquark condensate Δ (triangles) and the electron's chemical potential μ_e (circles) are plotted as functions of the quark's chemical potential μ. It is found that when chiral symmetry is restored, the color superconducting phase appears. In the color superconducting phase, it is found that both the diquark condensate Δ and the electron's chemical potential first increase with increasing μ, reach their maximum at about $\mu = 475$ MeV, then decrease rapidly with increasing μ, and the diquark condensate disappears at about $\mu = 535$ MeV. Comparing the diquark condensate Δ_0 with Δ, we can see that the diquark condensate Δ

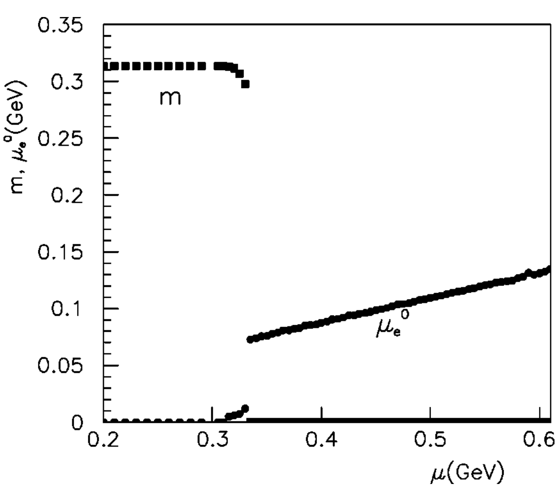

FIG. 2. The quark mass m (squares) and the electron's chemical potential μ_e^0 (circles) as functions of chemical potential μ in the case of neutral normal quark matter.

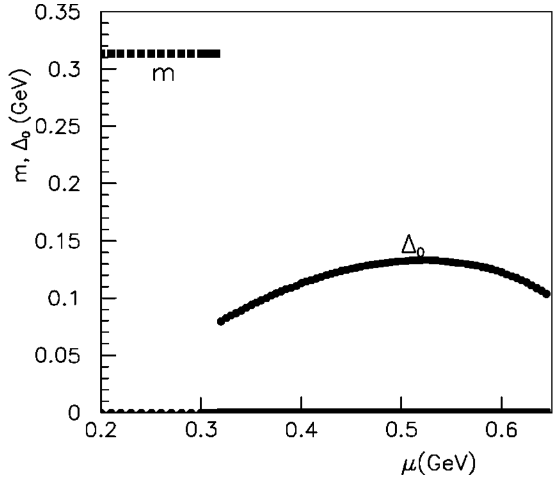

FIG. 3. The quark mass m (squares) and the diquark condensate Δ_0 (circles) as functions of chemical potential μ in the case of no charge neutrality on the color superconducting phase.

in the charge neutral superconducting phase has a much smaller value than Δ_0 in the charged superconducting phase; it means that the difference of the Fermi surfaces of the two pairing quarks reduces the magnitude of the diquark condensate Δ.

In Fig. 5 we show the chemical potential μ_{8c} as a function of μ, which ensures the system having zero net color charge T_8. It is found that μ_{8c} can be negative and positive, but its value is very small, only about several MeV. It means that for the same flavor quark u or d, the difference of the Fermi momenta between the third colored quark and the first two colored quarks can be neglected.

In order to explicitly see what happens to a color superconducting phase when charge neutrality condition is added, we compare the electron's chemical potential in the neutral normal quark matter μ_e^0 (light circles) and in the neutral superconducting phase μ_e (solid circles) in Fig. 6.

From Fig. 6 we find that the electron has a larger Fermi

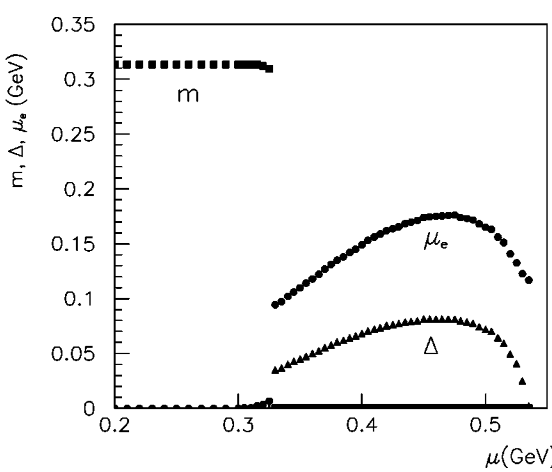

FIG. 4. The quark mass m (squares), the diquark condensate Δ (triangles), and the electron's chemical potential μ_e (circles) as functions of chemical potential μ in the case of considering charge neutrality on the color superconducting phase.

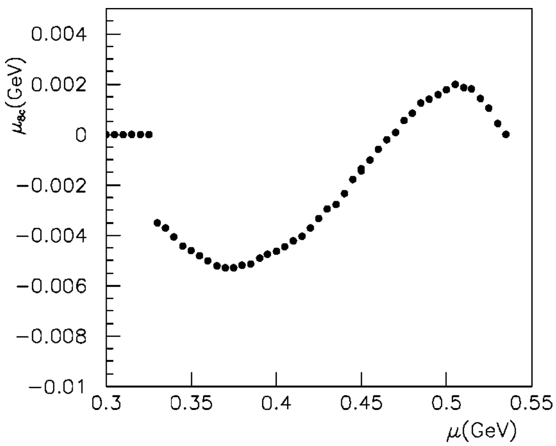

FIG. 5. The μ_{8c} as a function of chemical potential μ in the case of considering charge neutrality on the color superconducting phase.

surface in the neutral superconducting phase than that in the neutral normal quark matter. This is because in the superconducting phase, the isospin is mainly carried by unpaired quarks only, and μ_e has to be larger in order to get the same isospin density. μ_e is equal to μ_e^0 at about $\mu=535$ MeV, at which the diquark condensate disappears. We separate μ_e into two parts as $\mu_e=\mu_e^0+\delta\mu_e$, where $\delta\mu_e$ reflects the effect induced by the diquark condensate; comparing $\delta\mu_e$ (light squares) and Δ (solid squares) in Fig. 6, we can see that $\delta\mu_e$ increases when Δ increases and decreases when Δ decreases, and when $\Delta=0$, $\delta\mu_e=0$, too.

From Fig. 6 we can see a distinguished characteristic in the neutral superconducting phase, i.e., both the electron's chemical potential μ_e and the magnitude of the diquark condensate Δ first increase with increasing μ, then at about $\mu=475$ MeV, decrease with increasing μ. Now we try to understand this phenomena.

The magnitude of the diquark condensate is not only affected by μ, but also by $\delta\mu=\mu_e/2$, which describes the

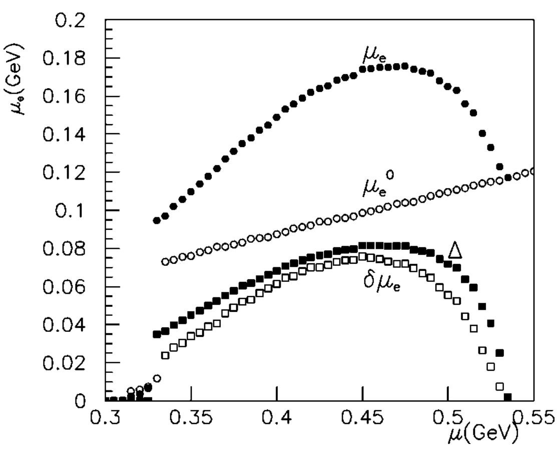

FIG. 6. The electron's chemical potential in the neutral normal quark matter μ_e^0 (light circles) and in the neutral superconducting phase μ_e (solid circles) as functions of chemical potential μ, and $\delta\mu_e=\mu_e-\mu_e^0$ (light squares) comparing with Δ (solid squares) as functions of μ.

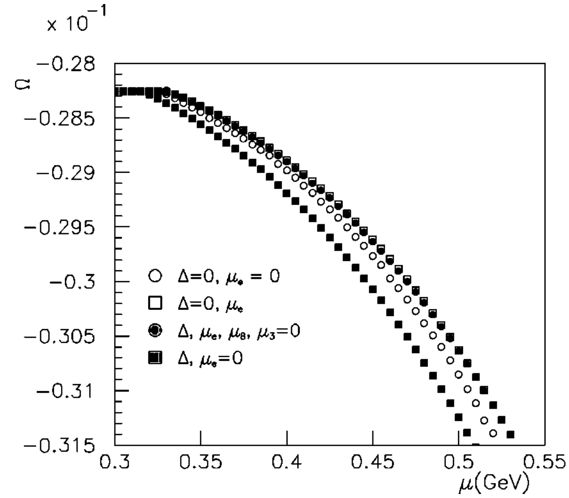

FIG. 7. The thermodynamic potentials as functions of chemical potential μ, the light squares and circles are for the cases with and without charge neutrality and without diquark condensate, and the dark circles and squares are for the cases with and without charge neutrality and with diquark condensate.

difference of the Fermi surface between the two paired quarks and reduces the diquark condensate. The diquark condensate first increases with increasing μ, which is the result of the increasing density of states. At about $\mu=475$ MeV, where $\mu_d=\mu+1/3\mu_e$ is about 530 MeV, the diquark condensate starts to be affected by the momentum cutoff Λ. Like that in the charged color superconducting phase, the diquark condensate stops increasing with μ, so does μ_e. However, when $\mu>475$ MeV, μ_e^0 keeps increasing with μ, which reduces the diquark condensate largely. This is why we see Δ decreases with increasing μ. Finally at a certain chemical potential $\mu=535$ MeV, where μ_d is about 580 MeV, the diquark condensate disappears. Because $\delta\mu_e$ decreases with the decrease of Δ, the total μ_e decreases in the chemical potential regime $\mu>475$ MeV.

In Fig. 7 we show the thermodynamic potentials in the four different cases as functions of chemical potential μ, the light squares and circles are for the cases of normal quark matter with and without charge neutrality, the dark circles and squares are for the cases of color superconducting phases with and without charge neutrality. It can be seen that in the chiral symmetric phase, the superconducting phase without the charge neutrality condition added has the lowest thermodynamic potential, and the normal quark matter without the charge neutrality condition added has the second lowest Ω. The neutral quark systems have higher thermodynamic potentials, but the neutral superconducting phase has a little bit lower Ω than that of the neutral quark matter in the chemical potential region $\mu<535$ MeV, and at $\mu=535$ MeV, the two Ωs coincide with each other, at which the diquark condensate in the neutral system disappears. Therefore, in the chiral symmetric phase, the stable state of the neutral system is the color superconducting phase for $<\mu<535$ MeV. At $\mu=535$ MeV, the stable phase is the normal neutral quark matter.

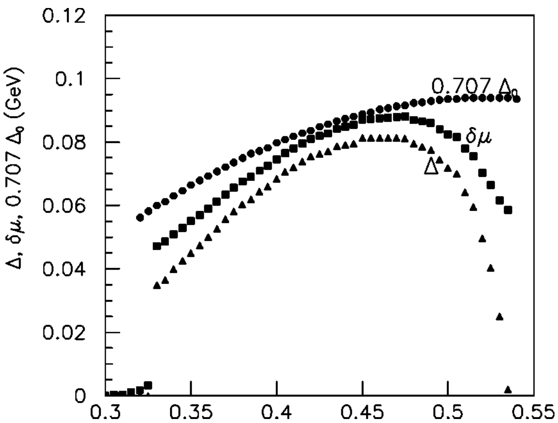

FIG. 8. $\delta\mu$ (squares), Δ (triangles), and $\Delta_0/\sqrt{2}$ (solid circles) as functions of chemical potential μ.

At last, we compare the magnitude of $\delta\mu$ (squares), Δ (triangles) and $\Delta_0/\sqrt{2}$ (solid circles) in Fig. 8 as functions of chemical potential μ. Sometimes, it was thought that $\delta\mu$ should be less than Δ for the formation of the BCS pair. In our numerical calculations, it is found that in the charge neutral color superconduting phase, $\delta\mu$ is always larger than Δ. Because $\delta\mu > \Delta$, from the expressions of number densities for u_1, u_2 and d_1, d_2, we can see that the two paired quarks have different number densities. In [7], it was argued that for the pairing between the two quark species with a chemical potential difference $2\delta\mu$, if $\delta\mu > \Delta_0/\sqrt{2}$, then the BCS pairing is not possible. From our numerical results, it is found that $\delta\mu$ is always less than $\Delta_0/\sqrt{2}$. However, the BCS pair still breaks at a certain chemical potential, at which the thermodynamic potential for the charge neutral color superconducting phase equals that of the charge neutral normal quark matter.

V. CONCLUSIONS

We investigated four different phases of the two-flavor quark system, i.e., the normal quark matter and color superconducting phases with and without the charge neutrality condition, in the framework of the SU(2) Nambu–Jona-Lasinio model. It has been found that the color superconducting phase without charge neutrality has the lowest thermodynamic potential, and the charge neutral systems have higher thermodynamic potentials. However, the BCS pairing always lowers the system's thermodynamic potential, i.e., for both normal or charge neutral systems, the superconducting phase always has the lower thermodynamic potential. Comparing with the BCS gap Δ_0 for the normal color superconducting phase, the BCS gap Δ in the charge neutral color superconducting phase has been largely reduced by the difference of the two paired quarks. The diquark condensate first increases with the increase of quark's chemical potential μ, then decreases rapidly and disappears at about $\mu = 535$ MeV, at which the thermodynamic potential for the charge neutral color superconducting phase equals that in the neutral normal quark matter.

As we mentioned in the Introduction, we did not consider the strange quark in the system, which should be considered and has been investigated self-consistently in [10]. Comparing our results with their results about the diquark gap Δ, the electron's chemical potential μ_e and the chemical potential μ_{8c} in 2SC, it can be found that the main difference lies in the chemical potential region $\mu > 400$ MeV. In [10], the diquark condensate in the neutral superconducting phase is largely reduced only in a small chemical potential region 370 MeV $< \mu <$ 400 MeV. μ_e first increases with μ then begins to decrease at about $\mu = 400$ MeV. As for μ_{8c}, the tendency also becomes different from our results at about $\mu = 400$ MeV. The reason lies in that, in [10], the strange quark involves in the system when $\mu > 400$ MeV. This shows that the effect of the charge neutrality condition on three- and two-flavor quark systems is quite different.

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